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Approach to group decision making based on determining the weights of experts by using projection method

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ABSTRACT

The aim of this paper is to present a new approach for determining weights of experts in the group decision making problems. Group decision making has become a very active research field over the last decade. Especially, the investigation to determine weights of experts for group decision making has attracted great interests from researchers recently and some approaches have been developed. In this paper, the weights of experts are determined in the group decision environment via projection method. First of all, the average decision of all individual decisions is defined as the ideal decision. After that, the weight of expert is determined by the projection of individual decision on the ideal decision. By using the weights of experts, all individual decisions are aggregate into a collective decision. Then an ideal solution of alternatives of the collective decision, expressed by a vector, is determined. Further, the preference order of alternatives are ranked in accordance with the projections of alternatives on the ideal solution. Comparisons with an extended TOPSIS method are also made. Finally, an example is provided to illustrate the developed approach.

1. Introduction

Decision making problem as one of the most important problems in all sciences is the process of finding the best option from all of the feasible alternatives. In many cases, the decision maker needs to take a decision based on multiple attributes to select an alternative from those feasible ones. Multiple attributes decision making (MADM) is an important part of modern decision science, which contains multiple decision attributes and multiple decision alternatives. The aim is to help the decision maker take all important objective and subjective criteria/attributes of the problem into consideration using a more explicit, rational and efficient decision process [1,2]. MADM has been extensively applied to various areas such as society, economics, military, management, etc., and has been receiving more and more attention over the last decades [3,4].

The increasing complexity of the engineering and management environment leads to benefit from a group of experts or decision makers to investigate all relevant aspects of decision making problems [5]. In the recent decade, some studies focused on MADM problems to provide reliable results and take into account the analysis of the experts instead of the analysis of a single expert. This makes that the multiple attributes group decision making (MAGDM) is attracting more and more attention in management, and has received a great deal of attention from researchers [6–12].

The MAGDM problems have three common characteristics: alternatives, multiple attributes with incommensurable units and multiple experts, in which the weights of experts play a very important role, how to determine the weights of experts will be an interesting and important research topic. At present, many methods have been proposed to determine the weights of experts. French [13] proposed a method to determine the relative importance of the group's members by using the

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influence relations, which may exist between the members. Theil [14] proposed a method based on the correlation concepts when the member's inefficacy is measurable. Keeney and Kirkwood [15] and Keeney [16] suggested the use of the interpersonal comparison to determine the scales constant values in an additive and weighted social choice function. Bodily [17] and Mirkin and Fishburn [18] proposed two approaches which use the eigenvectors method to determine the relative importance of the group's members. Brock [19] used a Nash bargaining based approach to estimate the weights of group members intrinsically. Ramanathan and Ganesh [20] proposed a simple and intuitively appealing eigenvector based method to intrinsically determine the weights of group members using their own subjective opinions. Martel and Ben Khélifa [21] proposed a method to determine the relative importance of group's members by using individual outranking indexes. Van den Honert [22] used the REMBRANDT system (multiplicative AHP and associated SMART model) to quantify the decisional power vested in each member of a group, based on subjective assessments by the other group members. Jabeur and Martel [23] proposed a procedure which exploits the idea of Zeleny [24] to determine the relative importance coefficient of each member. Chen and Fan [25] proposed a factor score method for obtaining a ranking of the assessment levels of experts in group-decision analysis. By using the deviation measures between additive linguistic preference relations, Xu [26] gave some straightforward formulas to determine the weights of experts. Chen and Fan [27] studied a method for the ranking of experts according to their levels in group decision. Yue [28,29] presented an approach for group decision making based on determining weights of DMs using TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) [30]. Recently, Yue [31] developed a new approach for measuring the decision makers' weights in group decision making setting based on distance measure, in which the decision information is expressed in interval-valued intuitionistic fuzzy numbers.

The existing approaches dealing with the weights of experts can be divided into two categories: the subjective preference information of expert is represented by Saaty's multiplicative preference relation [12] and others, including subjective and objective preference information taken the form of real numbers [29], interval numbers [28], language [26,27], and other [25,31].

Most of the existing approaches are to take the form of Saaty's multiplicative preference relation. The disadvantages of these approaches are that subjectivity of experts is too strong and the procedure dealing with the weights of experts is very complicated. To resolve these problems, by using the TOPSIS, Yue [28,29,31] developed some methods for determining weights of experts.

In this study, we propose a straightforward and practical method to deriving the weights of experts and ranking the preference order of alternatives based on projection method [4,32–35]. Projection method is used twice to the developed approach in this paper, which is first used to determine the weights of experts, and second used to rank the preference order of alternatives.

The reminder of this paper is organized as follows: in the next section, briefly introduces the projection method. In Section 3, we present an algorithm for MAGDM based on determining the weights of experts using projection method. In Section 4, we make some comparisons between the presented method and an extended TOPSIS method. In Section 5, we illustrate our proposed algorithmic method with an example. The final section concludes.

2. Projection method

Definition 1 [4]. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ be a vector, then

$$|\alpha| = \sqrt{\sum_{j=1}^{n} \alpha_j^2} \tag{1}$$

is called the module of vector α.

Definition 2 [4]. Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ be two vectors, then

$$\alpha\beta = \sum_{j=1}^{n} \alpha_j \beta_j \tag{2}$$

is called the inner product between α and β.

Through a combination of Eqs. (1) and (2), we have the concept of projection between two vectors as follows:

Definition 3 ([4,32,33]). Let $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ be two vectors, then

$$Prj_\beta(\alpha) = |\alpha| cos(\alpha, \beta) = |\alpha| \frac{\alpha\beta}{|\alpha||\beta|} = \frac{\alpha\beta}{|\beta|} \tag{3}$$

is called the projection of the vector α on the β.

The projection can be illustrated in Fig. 1.

In general, the bigger the value of $Prj_\beta(\alpha)$, the more the degree of the vector α approaching to the vector β.

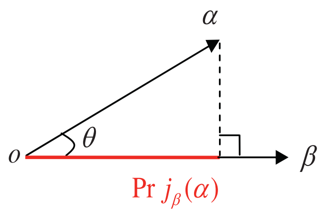

Fig. 1. Projection of vector α on β.

Similarly to the projection between vectors, in the following, we introduce the projection between matrices.

Definition 4. Let $A = (a_{ij})_{m\times n}$ and $B = (b_{ij})_{m\times n}$ be two matrices, then

$$Prj_B(A) = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}b_{ij}}{\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} b_{ij}^2}} \tag{4}$$

is called the projection of the matrix A on the matrix B.

Similarly, the larger the value of $Prj_B(A)$, the more the degree of the matrix A approaching to the matrix B.

3. The presented algorithm

3.1. Characterization of MAGDM problem

For convenience, in this paper, let $M = \{1,2,\ldots,m\}$, $N = \{1,2,\ldots,n\}$ and $T = \{1,2,\ldots,t\}$; $i \in M$, $j \in N$, $k \in T$. And let $\{A_1,A_2,\ldots,A_m\}$ $(m \geqslant 2)$ be a discrete set of m feasible alternatives, $\{u_1,u_2,\ldots,u_n\}$ be a finite set of attributes, $(w_1,w_2,\ldots,w_n)^T$ be the weight vector of attributes, such that $0 \leqslant w_j \leqslant 1$ and $\sum_{j=1}^{n} w_j = 1$; $\{d_1,d_2,\ldots,d_t\}$ be a group of experts, and $(\lambda_1,\lambda_2,\ldots,\lambda_t)^T$ be the weight vector of experts, where $\lambda_k \geqslant 0$, $\sum_{k=1}^{n}\lambda_k = 1$.

A MAGDM problem can be described as follows:

Let

$$X_k = \left(x_{ij}^{(k)}\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} x_{11}^{(k)} & x_{12}^{(k)} & \cdots & x_{1n}^{(k)} \\ x_{21}^{(k)} & x_{22}^{(k)} & \cdots & x_{2n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1}^{(k)} & x_{m2}^{(k)} & \cdots & x_{mn}^{(k)}, \end{bmatrix}\end{array} \tag{5}$$

$$\left(w_1^{(k)}, w_2^{(k)}, \ldots, w_n^{(k)}\right)^T, \quad k \in T$$

be the decision matrix and attributes' weight vector given by kth expert. In general, there are benefit attributes and cost attributes in the MADM problems. In order to measure all attributes in dimensionless units and facilitate inter-attribute comparisons, we introduce the following Eqs. (7) and (8) [29] to normalize each attribute value $x_{ij}^{(k)}$ in decision matrix X_k into a corresponding element $r_{ij}^{(k)}$ in normalized decision matrix R_k given by Eq. (6).

$$R_k = \left(r_{ij}^{(k)}\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array}\begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} r_{11}^{(k)} & r_{12}^{(k)} & \cdots & r_{1n}^{(k)} \\ r_{21}^{(k)} & r_{22}^{(k)} & \cdots & r_{2n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1}^{(k)} & r_{m2}^{(k)} & \cdots & r_{mn}^{(k)} \end{bmatrix}\end{array}, \quad k \in T, \tag{6}$$

where

$$r_{ij}^{(k)} = \frac{x_{ij}^{(k)}}{\sqrt{\sum_{i=1}^{m}(x_{ij}^{(k)})^2}}, \quad \text{for benefit attribute } u_j, \quad i \in M, \quad j \in N, \ k \in T \tag{7}$$

and

$$r_{ij}^{(k)} = 1 - \frac{x_{ij}^{(k)}}{\sqrt{\sum_{i=1}^{m}(x_{ij}^{(k)})^2}}, \quad \text{for cost attribute } u_j, \quad i \in M, \quad j \in N, \ k \in T. \tag{8}$$

For the attributes' weight vector $\left(w_1^{(k)}, w_2^{(k)}, \ldots, w_n^{(k)}\right)^T$ given by kth DM, we can construct the weighted normalized decision matrix as

$$Y_k = \left(w_j^{(k)} r_{ij}^{(k)}\right)_{m\times n} = \left(y_{ij}^{(k)}\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} y_{11}^{(k)} & y_{12}^{(k)} & \cdots & y_{1n}^{(k)} \\ y_{21}^{(k)} & y_{22}^{(k)} & \cdots & y_{2n}^{(k)} \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1}^{(k)} & y_{m2}^{(k)} & \cdots & y_{mn}^{(k)} \end{bmatrix} \end{array} \quad k \in T. \tag{9}$$

As described above, suppose that a process of MAGDM needs t experts, each expert may provide his/her preferences over alternatives with respect to attributes, all provided preference values can be conveniently contained in a matrix, and Y_1, Y_2, ..., Y_t are the decision matrices of t experts, and Y^* is the ideal decision of $Y_1, Y_2, \ldots, Y_t$. The basic idea of this algorithm is that the more the degree of the decision matrix Y_k approaching to the ideal decision Y^*, the bigger the weight of kth expert. That is to say, the larger the value of the projection of the decision matrix Y_k on the ideal decision Y^*, the bigger the weight of kth expert. The problem is how to obtain the ideal decision?

According to the individual decision $Y_k = \left(y_{ij}^{(k)}\right)_{m\times n}$ in Eq. (9), we can get the average decision of Y_k $(k \in T)$ as follows:

$$Y^* = \left(y_{ij}^*\right)_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} y_{11}^* & y_{12}^* & \cdots & y_{1n}^* \\ y_{21}^* & y_{22}^* & \cdots & y_{2n}^* \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1}^* & y_{m2}^* & \cdots & y_{mn}^* \end{bmatrix} \end{array} \tag{10}$$

where $Y^* = \frac{1}{t}\sum_{k=1}^t Y_k$, and $y_{ij}^* = \frac{1}{t}\sum_{k=1}^t y_{ij}^{(k)}$ $(i \in M,\ j \in N)$.

Inspired by the literature in [28,29], in compromise sense, we define $Y^* = \left(y_{ij}^*\right)_{m\times n}$ as the ideal decision of all individual decision Y_k $(k \in T)$ in Eq. (9). In this sense, the more the degree that Y_k is closer to the Y^*, the better the decision Y_k.

In order to measure the decision level of each expert, we can calculate the projection of each individual decision matrix Y_k $(k \in T)$ on ideal decision Y^*. By Eq. (4), the projection can be given as

$$Proj_{Y^*}(Y_k) = \frac{\sum_{i=1}^m \sum_{j=1}^n y_{ij}^{(k)} y_{ij}^*}{\sqrt{\sum_{i=1}^m \sum_{j=1}^n \left(y_{ij}^*\right)^2}}, \quad k \in T. \tag{11}$$

Also, the more the degree that individual decision Y_k is closer to the Y^*, the bigger the projection $Proj_{Y^*}(Y_k)$, and then we shall assign higher weight to kth expert.

In order to get the weights of experts by these projections, we can make the following transformation

$$\lambda_k = \frac{Proj_{Y^*}(Y_k)}{\sum_{k=1}^t Proj_{Y^*}(Y_k)}, \quad k \in T, \tag{12}$$

where, obviously, $\lambda_k \geqslant 0$ and $\sum_{k=1}^t \lambda_k = 1$, for all $k \in T$.

For the weight vector $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)^T$ of experts, we can calculate the collective decision by

$$Y = \sum_{k=1}^t \lambda_k Y_k = (y_{ij})_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ y_{m1} & y_{m2} & \cdots & y_{mn} \end{bmatrix} \end{array} \tag{13}$$

where $y_{ij} = \sum_{k=1}^t \lambda_k y_{ij}^{(k)}$ $(i \in M, j \in N)$.

Here, we give the ideal solution of alternatives, and then to find the most satisfactory one(s) according to the traditional TOPSIS method, which the hierarchical structure is illustrated in Fig. 2.

Definition 5 [36]. The vector $y^+ = (y_1^+, y_2^+, \ldots, y_n^+)$ is called the ideal solution of alternatives $A_1, A_2, \ldots, A_m$, if

$$y_j^+ = \max_{1\leqslant i\leqslant m}\{y_{ij}\}, \quad j \in N, \tag{14}$$

where $y_i = (y_{i1}, y_{i2}, \ldots, y_{in})$ $(i \in M)$ is the ith row-vector of Y in Eq. (13).

Then, by Eq. (3), the projections of alternatives $A_1, A_2, \ldots, A_m$ on the ideal solution y^+ is shown as follows

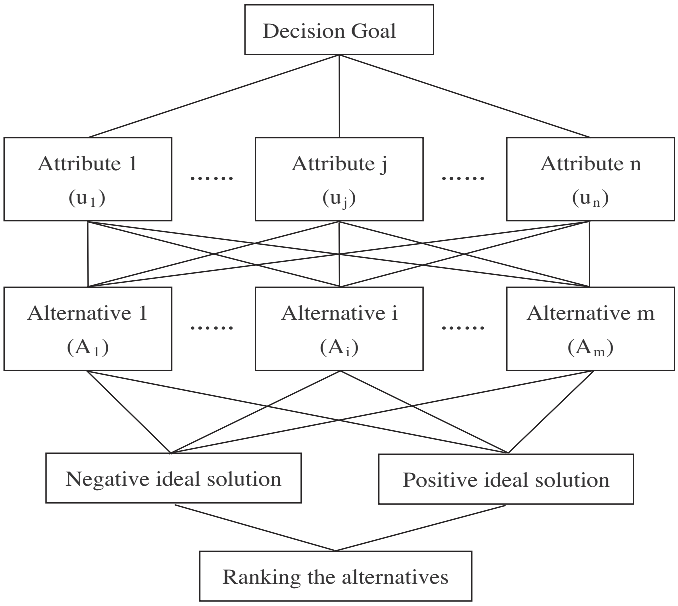

Fig. 2. Hierarchical structure of traditional TOPSIS.

$$Prj_{y^+}(y_i) = \frac{\sum_{j=1}^{n} y_{ij} y_j^+}{\sqrt{\sum_{j=1}^{n} \left(y_j^+\right)^2}}, \quad i \in M, \tag{15}$$

where $y_i = (y_{i1}, y_{i2}, \ldots, y_{in})$ $(i \in M)$ is the ith row-vector of Y in Eq. (13).

Obviously, the more the degree that y_i is closer to the y^+, the bigger the projection $Prj_{y^+}(y_i)$, and thus the better the alternative A_i. Therefore, by the values of projections $Prj_{y^+}(y_i)$, we can rank all alternatives and find the most desirable one (s).

3.2. The presented algorithm

In summary, an algorithm for MAGDM, based on determining the expert's weight, using projection method, can be shown as the following steps.

Step 1. Each expert provides his/her individual decision matrix X_k $(k \in T)$ and weight vector $\left(w_1^{(k)}, w_2^{(k)}, \ldots, w_n^{(k)}\right)^T$ $(k \in T)$ of attributes by using Eq. (5).
Step 2. Utilize the Eq. (7) and/or (8) to normalize $X_k (k \in T)$ into R_k $(k \in T)$ in Eq. (6).
Step 3. Utilize the Eq. (9) to calculate the weighted normalized decision matrixes Y_k $(k \in T)$.
Step 4. Determine the ideal decision Y^* for all individual decisions by using Eq. (10).
Step 5. Calculate the projection $Proj_{Y^*}(Y_k)$ of matrix Y_k $(k \in T)$ on the ideal decision Y^* by using Eq. (11).
Step 6. Determine the weight of expert by using Eq. (12).
Step 7. Calculate the collective decision by using Eq. (13).
Step 8. Calculate the ideal solution of alternatives by using Eq. (14).
Step 9. Calculate the projection of alternative by using Eq. (15).
Step 10. Rank the preference order of alternatives in accordance with their projections, i.e., the bigger the projection, the better the alternative.

4. Comparing the projection method with an extended TOPSIS

TOPSIS [37–40], one of the major MADM techniques, was first developed by Hwang and Yoon [30]. It ranks the alternatives according to their distances from the positive and the negative ideal solutions, i.e. the best alternative has simultaneously the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS). The PIS is identified with a ''hypothetical alternative'' that has the best values for all considered attribute whereas the NIS is identified with a ''hypothetical alternative'' that has the worst attribute values (see Fig. 2).

In this part, the proposed method is compared with the extended TOPSIS introduced by Yue [29] for determining the weights of experts. The results are shown in Table 1 and the differences are illustrated in Figs. 3 and 4.

Table 1
Comparison the projection method with the extended TOPSIS method.

Characteristics	Extended TOPSIS	Projection
Evaluation objective	Selection and ranking of a number of experts	Selection and ranking of a number of experts
No. of experts	More than one	More than one
No. of ideal decisions	Three	One
Key decision	Relative closeness	Projection
Core factors	The distances from each individual decision to ideal decisions	Both distance and angle from each individual decision to ideal decision
Goal(s)	Both maximum profit and minimum risk/regret	Maximum profit
Final decision	Ranking of a number of alternatives	Ranking of a number of alternatives

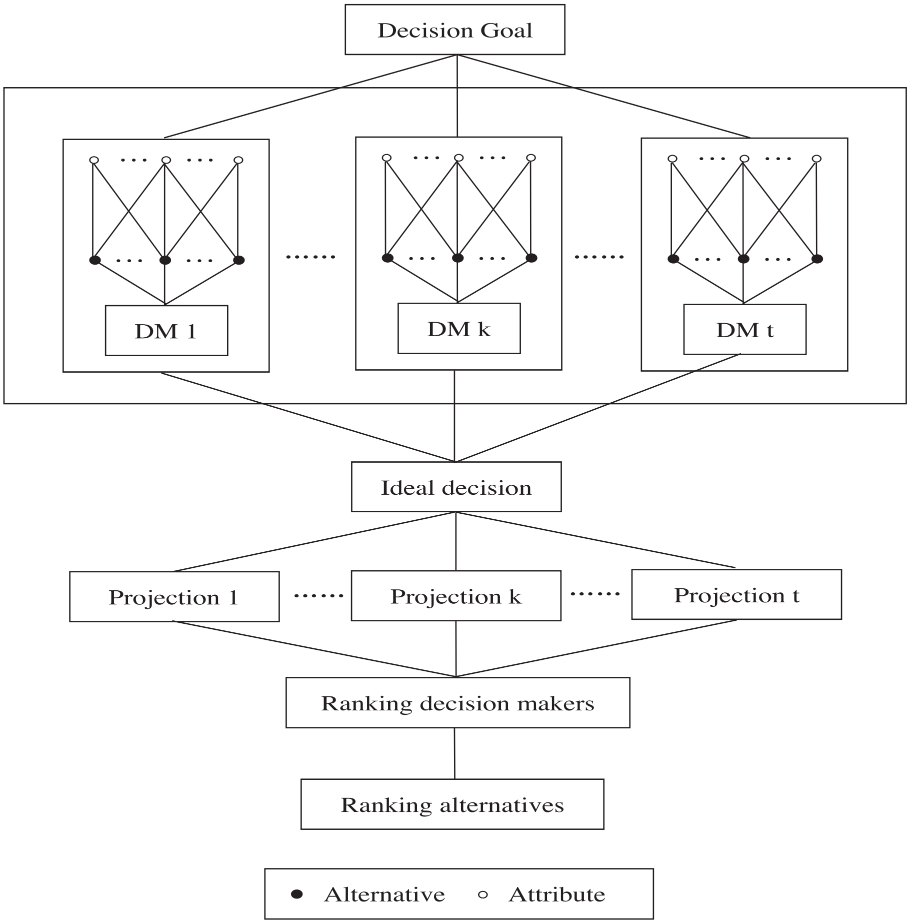

Fig. 3. Hierarchical structure of the proposed projection method.

Table 1 illustrates the differences and similarities between the proposed method in this paper and the extended TOPSIS introduced by Yue [29]. The proposed method uses projection of individual decision on ideal decision to determining the weights of experts; whereas the extended TOPSIS uses Euclides distance to determining the relative closeness, then obtains the weights of experts; The proposed method is simple since only one ideal decision is used to determining the projections of individual decisions, then obtains the weights of experts.

Figs. 3 and 4 show the hierarchical structure of the projection method proposed in this paper and the extended TOPSIS method introduced by Yue [29], respectively.

Table 1 and Fig. 3 show that the weight of each expert is determined by his/her own decision. The more the degree that the individual decision is closer to the ideal decision, the better the decision, furthermore, the bigger the weight of expert. The best decision is done by a pseudo-expert, whose decision is the ideal decision (the average of all individual decisions).

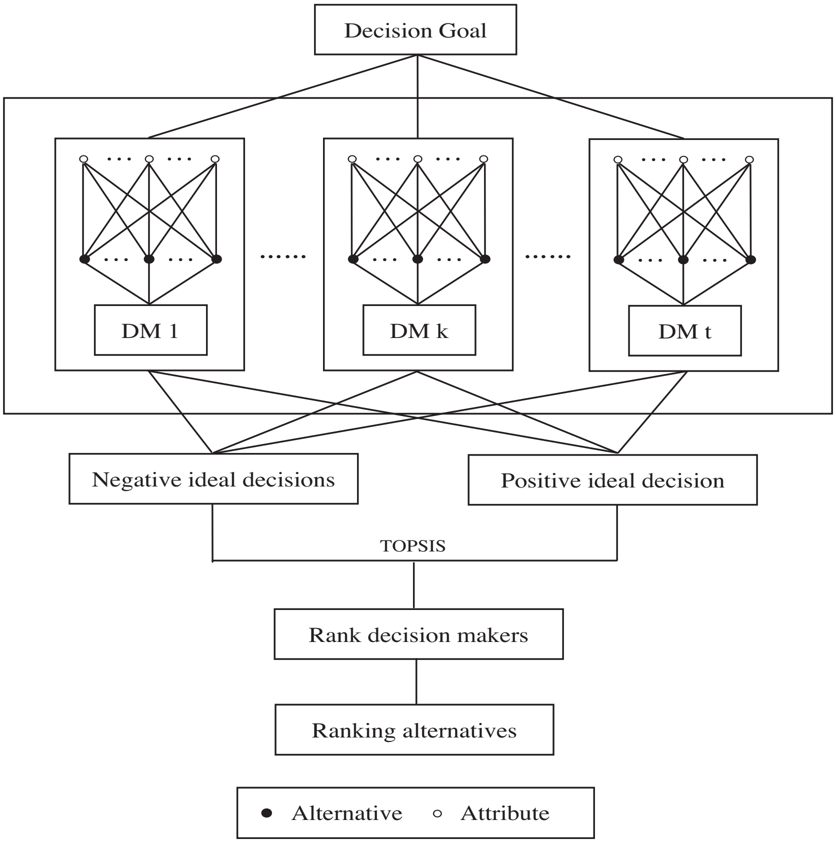

Fig. 4. Hierarchical structure of the extended TOPSIS method.

From this point of view, The more the degree that the individual decision is closer to the average of all individual decisions, the better the representing majority in mean sense. Otherwise, the proposed method assigns low weights to those "false" or "biased" ones.

TOPSIS method is suitable for cautious (risk avoider) expert(s), because the expert(s) might like to have a decision which not only makes as much profit as possible, but also avoids as much risk as possible [41]. The projection method in this paper assigns high weights to those experts if the experts want to have maximum profit, and the risk of the decisions is less important for him.

Projection method has the following advantages: (i) Straightforward idea. The basic idea of projection method is rather straightforward. The projection of a vector/matrix on another vector/matrix, in a sense, is a approaching measure of a vector/matrix to another. (ii) Simple procedure. Comparing the projection method with the extended TOPSIS method, it is clear that the extended TOPSIS method needs the positive ideal decision and negative ideal decisions as two "reference" points, however, the projection method only needs the positive ideal decision as a "reference" point. Thus, it is simple. (iii) Comprehensive consideration. The projective method not only considers the distances between vectors/matrices, so, its consideration is comprehensive. And (iv) Relaxed conditions. The method has no limits for the data distribution, the number of indexes and the sample size. It can not only be suitable for handling the fewer alternatives, fewer indexes and small sample, but also be suitable for handling the multi-alternative, multi-index and large sample.

5. Illustrative example

In the following, an instance (adapted from [38]) is provided to illustrate the proposed approach.

Example. A human resources selection example.

A local chemical company tries to recruit an on-line manager. The company's human resources department provides some relevant selection tests as the benefit attributes to be evaluated. These objective tests include knowledge tests (language test, professional test and safety rule test), skill tests (professional skills and computer skills). After these objective tests, there are 17 qualified candidates (as alternatives marked by $A_1, A_2, \ldots, A_{17}$, or briefly marked by $1, 2, \ldots, 17$) on the list for the selection. Then four experts (marked by d_1, d_2, d_3, d_4) are responsible for the selection from among them based

Table 2
Decision matrixes of example–subjective attributes.

No. of candidates	X_1		X_2		X_3		X_4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	80	75	85	80	75	70	90	85
2	65	75	60	70	70	77	60	70
3	90	85	80	85	80	90	90	95
4	65	70	55	60	68	72	62	72
5	75	80	75	80	50	55	70	75
6	80	80	75	85	77	82	75	75
7	65	70	70	60	65	72	67	75
8	70	60	75	65	75	67	82	85
9	80	85	95	85	90	85	90	92
10	70	75	75	80	68	78	65	70
11	50	60	62	65	60	65	65	70
12	60	65	65	75	50	60	45	50
13	75	75	80	80	65	75	70	75
14	80	70	75	72	80	70	75	75
15	70	65	75	70	65	70	60	65
16	90	95	92	90	85	80	88	90
17	80	85	70	75	75	80	70	75

Note: (1) There are four experts selected for the evaluation. (2) There are a total of 17 candidates for evaluation. (3) All listed attributes are benefit attributes.

on subjective tests. The basic data of subjective attributes, including panel interview and 1-on-1 interview tests (only quantitative information here) for the decision, are listed in Table 2.

The following results/processes are programmed/performed with software MATLAB.

Following the suggested steps in Section 3, we will construct the normalized decision matrices for Table 2. Since all listed attributes are benefit attributes, by Eq. (7), we first normalize Table 2 into Table 3 according to Step 2. Table 3 includes 4 normalized decision matrices R_1, R_2, R_3, and R_4.

In addition, the weights of attributes, provided by experts, are shown in Table 4.

By Step 3, the columns of Table 3 can respectively be multiplied by the associated weights given by experts in Table 4. Then, the weighted normalized decision results are shown in Table 5.

The ideal decision Y^{*}, by Step 4, is shown in Table 6.

By Step 5, we can calculate the projection of each weighted normalized decision matrix on the ideal decision, which are summarized in Table 7.

Further, we can calculate the weights of experts by Step 6 and experts' ranking, which are organized in Table 7. The final experts' priority ranking produced by the projection method in this paper is as

Table 3
Normalized decision matrixes.

No.	R_1		R_2		R_3		R_4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	0.2624	0.2416	0.2747	0.2565	0.2552	0.2297	0.2988	0.2683
2	0.2132	0.2416	0.1939	0.2245	0.2382	0.2526	0.1992	0.2209
3	0.2952	0.2738	0.2585	0.2726	0.2722	0.2953	0.2988	0.2998
4	0.2132	0.2255	0.1777	0.1924	0.2314	0.2362	0.2058	0.2272
5	0.2460	0.2577	0.2424	0.2565	0.1702	0.1805	0.2324	0.2367
6	0.2624	0.2577	0.2424	0.2726	0.2620	0.2690	0.2490	0.2367
7	0.2132	0.2255	0.2262	0.1924	0.2212	0.2362	0.2224	0.2367
8	0.2296	0.1933	0.2424	0.2084	0.2552	0.2198	0.2722	0.2683
9	0.2624	0.2738	0.3070	0.2726	0.3063	0.2789	0.2988	0.2904
10	0.2296	0.2416	0.2424	0.2565	0.2314	0.2559	0.2158	0.2209
11	0.2296	0.2416	0.2004	0.2084	0.2042	0.2133	0.2158	0.2209
12	0.1968	0.2094	0.2101	0.2405	0.1702	0.1969	0.1494	0.1578
13	0.2460	0.2416	0.2585	0.2565	0.2212	0.2461	0.2324	0.2367
14	0.2624	0.2255	0.2424	0.2309	0.2722	0.2297	0.2490	0.2367
15	0.2296	0.2094	0.2424	0.2245	0.2212	0.2297	0.1992	0.2051
16	0.2952	0.3061	0.2973	0.2886	0.2893	0.2625	0.2922	0.2840
17	0.2624	0.2738	0.2262	0.2405	0.2552	0.2625	0.2324	0.2367

Note: (1) There are four experts selected for the evaluation. (2) There are a total of 17 candidates for evaluation.

Table 4
Attributes' weights given by experts.

Attributes	The weights of the group			
	d_1	d_2	d_3	d_4
Panel interview	0.5243	0.4574	0.4160	0.4503
1-on-1 interview	0.4757	0.5426	0.5840	0.5497

Note: There are four experts selected for the evaluation.

Table 5
Weighted normalized decision matrixes.

No.	Y_1		Y_2		Y_3		Y_4	
	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview	Panel interview	1-on-1 interview
1	0.1376	0.1149	0.1256	0.1392	0.1062	0.1341	0.1345	0.1475
2	0.1118	0.1149	0.0887	0.1218	0.0991	0.1475	0.0897	0.1214
3	0.1548	0.1303	0.1182	0.1479	0.1133	0.1724	0.1345	0.1648
4	0.1118	0.1073	0.0813	0.1044	0.0963	0.1380	0.0927	0.1249
5	0.1290	0.1226	0.1109	0.1392	0.0708	0.1054	0.1046	0.1301
6	0.1376	0.1226	0.1109	0.1479	0.1090	0.1571	0.1121	0.1301
7	0.1118	0.1073	0.1035	0.1044	0.0920	0.1380	0.1002	0.1301
8	0.1204	0.0920	0.1109	0.1131	0.1062	0.1284	0.1226	0.1475
9	0.1376	0.1303	0.1404	0.1479	0.1274	0.1629	0.1345	0.1596
10	0.1204	0.1149	0.1109	0.1392	0.0963	0.1495	0.0972	0.1214
11	0.0860	0.0920	0.0916	0.1131	0.0849	0.1245	0.0972	0.1214
12	0.1032	0.0996	0.0961	0.1305	0.0708	0.1150	0.0673	0.0867
13	0.1290	0.1149	0.1182	0.1392	0.0920	0.1437	0.1046	0.1301
14	0.1376	0.1073	0.1109	0.1253	0.1133	0.1341	0.1121	0.1301
15	0.1204	0.0996	0.1109	0.1218	0.0920	0.1341	0.0897	0.1128
16	0.1548	0.1456	0.1360	0.1566	0.1203	0.1533	0.1316	0.1561
17	0.1376	0.1303	0.1035	0.1305	0.1062	0.1533	0.1046	0.1301

Note: (1) There are four experts selected for the evaluation. (2) There are a total of 17 candidates for evaluation.

Table 6
Ideal decision of all individual decisions.

No. of candidates	Panel interview	1-on-1 interview
1	0.1260	0.1339
2	0.0973	0.1264
3	0.1302	0.1539
4	0.0955	0.1186
5	0.1038	0.1243
6	0.1174	0.1394
7	0.1019	0.1199
8	0.1150	0.1202
9	0.1350	0.1502
10	0.1062	0.1313
11	0.0899	0.1128
12	0.0843	0.1080
13	0.1110	0.1320
14	0.1185	0.1242
15	0.1032	0.1171
16	0.1357	0.1529
17	0.1130	0.1360

Table 7
Projections, weights and ranking.

Experts	$Proj_{Y^*}(Y_k)$	λ_k	Ranking
d_1	0.7012	0.2478	4
d_2	0.7079	0.2502	3
d_3	0.7122	0.2517	1
d_4	0.7084	0.2503	2

Table 8
Collective assessment of 17 candidates.

No. of candidates	Panel interview	1-on-1 interview	Projections	Ranking
1	0.1259	0.1340	0.1838	4
2	0.0973	0.1265	0.1592	12
3	0.1301	0.1539	0.2015	3
4	0.0955	0.1187	0.1522	15
5	0.1037	0.1243	0.1618	11
6	0.1173	0.1395	0.1822	5
7	0.1018	0.1200	0.1573	13
8	0.1150	0.1203	0.1663	10
9	0.1350	0.1502	0.2019	2
10	0.1061	0.1313	0.1687	9
11	0.0899	0.1128	0.1441	16
12 (#)	0.0843	0.1080	<u>0.1367</u>	17
13	0.1109	0.1320	0.1724	7
14	0.1184	0.1243	0.1715	8
15	0.1032	0.1171	0.1561	14
16 (*)	0.1356	0.1529	<u>0.2044</u>	1
17	0.1129	0.1361	0.1767	6
Ideal solution	0.1356	0.1539		

Note: "*" and "#" mark the first and the last candidate, respectively. The underlined values denote the projection values of collective assessment of the first and the last candidate, respectively.

$$d_3 \succ d_4 \succ d_2 \succ d_1.$$

The 3rd column of Table 7 has illustrated that the vector $(0.2478, 0.2502, 0.2517, 0.2503)^T$ is weight vector of experts. By Step 7, we can aggregate four individual decisions in Table 5 into the collective decision, which is shown in the columns 2 and 3 of Table 8. The ideal solution of alternatives is calculated by Step 8, which is shown also in Table 8. The projections and ranking of alternatives are calculated by Steps 9 and 10, respectively, which are shown in columns 4 and 5 of Table 8, respectively. Table 8 shows that the 16th candidate is ranked first, and the 12th candidate is ranked last.

6. Conclusions

Many practical problems are often characterized by MAGDM problems. Evaluating decision levels of experts is an important research topic in group decision making. In this paper, we developed an approach for determining weights of experts in a group decision environment based on projection method. The proposed method is straightforward and can be performed on computer easily. As a future work, this paper should be extended to support situations where the information is in other forms, e.g., interval numbers, linguistic variables or fuzzy numbers.

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